Name: ______________________________

Instructor: __________________________

Math 10170, Practice Exam I
February 22, 2016

- The Honor Code is in effect for this examination. All work is to be your own.
- You may use your Calculator.
- The exam lasts for 50 minutes.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 12 pages of the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)

Please do NOT write in this box.

Multiple Choice	
8.	
9.	
10.	
11.	
12.	
Total	

Name: ______________________________

Instructor: __________________________

Multiple Choice

1.(6 pts.) Twenty members of a lacrosse squad are trying to decide what type of food to order after their game. Each member of the group has listed their preferences and the results are shown in the table below. The group will use the Borda Method (average rank) to decide which type of food to offer.

# of Voters

	5	**7**	**8**
Sushi	1	2	3
Hamburgers	2	1	1
Hot Dogs	3	3	2
Sandwiches	4	4	4
Pizza	5	5	5

The winner using the Borda Method is: .

(a) Sushi (b) Hot Dogs (c) Sandwiches

(d) Pizza (e) Hamburgers

2.(6 pts.) Consider the following matrices:

$$A = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad B = \begin{pmatrix} 0 & 2 \\ 4 & 1 \end{pmatrix} \quad C = \begin{pmatrix} 1 \\ 2 \end{pmatrix}.$$

Which of the following matrices is equal to $BA - C$?

(a) $\begin{pmatrix} 11 \\ -2 \end{pmatrix}$. (b) $\begin{pmatrix} 3 \\ 11 \end{pmatrix}$. (c) $\begin{pmatrix} 13 \\ 2 \end{pmatrix}$. (d) $\begin{pmatrix} 1 \\ 7 \end{pmatrix}$. (e) $\begin{pmatrix} 4 \\ 7 \end{pmatrix}$.

Name: ___________________________

Instructor: ________________________

3.(6 pts.) There are four dorms competing in the Notre Dame Spring Olympics. There are 10 events golf, rugby, 100 meter race, 5K race, darts, snooker, karate, boxing, soccer and Irish dancing. Each dorm has participants in each event. The table below shows the results for 2013 for the ten events (with first in an event denoted by 1).

Number of Events

	1	**2**	**1**	**1**	**2**	**3**
Morrisey Hall	1	4	2	3	2	4
Walsh Hall	2	3	3	1	1	3
Lyons Hall	4	1	1	4	4	1
O'Neill Hall	3	2	4	2	3	2

Each year "The Olympic Cup" is awarded one of the dorms based on their overall performance. If a Condorcet winner exists, the Olympic cup is awarded to that dorm, otherwise a Condorcet completion process is used to decide the winner. Which of the following is true?

(a) Walsh is the Condorcet winner

(b) Lyons is the Condorcet winner

(c) O'Neill is the Condorcet winner

(d) Morrisey is the Condorcet winner

(e) There is no Condorcet winner

Name: ____________________________

Instructor: ________________________

4.(6 pts.) An athlete is planning a diet for a twelve week training plan. She has a prescribed balance of carbohydrates, protein and fat for each meal. The athlete will three foods for tomorrow's breakfast.

One ounce of Food 1 has 30% of the required carbohydrates, 20% of the required protein and 0.1% of the required fat.

One ounce of Food 2 has 10% of the required carbohydrates, 40% of the required protein and 0% of the required fat.

One ounce of Food 3 has 0% of the required carbohydrates, 0% of the required protein and 50% of the required fat.

Let x denote the number of ounces of food 1 she will include, let y denote the number of ounces of food 2 she will include and let z denote the number of ounces of food 3 she will include. if she aims to have exactly 100% of the prescribed quantities of carbohydrates, protein and fat in her breakfast, which system of equations must she solve:

(a) $$\begin{aligned} 30x + 10y + 0z &= 100 \\ 20x + 40y + 0z &= 100 \\ 0.1x + 0y + 50z &= 100 \end{aligned}$$

(b) $$\begin{aligned} 30x + 20y + 0.1z &= 100 \\ 10x + 40y + 0z &= 100 \\ 0x + 0y + 50z &= 100 \end{aligned}$$

(c) $$\begin{aligned} 3x + 1y + 0z &= 100 \\ 2x + 4y + 0z &= 100 \\ 0.01x + 0y + 5z &= 100 \end{aligned}$$

(d) $$\begin{aligned} 30x + 10y + 0z &= 100 \\ 20x + 50y + 0z &= 100 \\ 0.1x + 0y + 40z &= 100 \end{aligned}$$

(e) $$\begin{aligned} 30x + 0y + 10z &= 100 \\ 20x + 0y + 40z &= 100 \\ 0.1x + 50y + 0z &= 100 \end{aligned}$$

Name: ______________________________

Instructor: __________________________

5.(6 pts.) The following table shows the results of a round robin in progress (up to Feb. 24 2014 the Six Nations Championship in Rugby).

	Ireland	England	Wales	Scotland	France	Italy	P-D
Ireland		10-13	26-3	28-6			42
England	13-10			20-0	24-26		21
Wales	3-26				27-6	23-15	6
Scotland	6-28	0-20				21-20	-41
France		26-24	6-27			30-10	1
Italy			15-23	20-21	10-30		-29

Which of the following matrix equations must be solved in order to find the Massey Ratings (keeping the same ordering of the teams as above)?

(a) $$\begin{pmatrix} 5 & -1 & -1 & -1 & 0 & 0 \\ -1 & 5 & 0 & -1 & -1 & 0 \\ -1 & 0 & 5 & 0 & -1 & -1 \\ -1 & -1 & 0 & 5 & 0 & -1 \\ 0 & -1 & -1 & 0 & 5 & -1 \\ 0 & 0 & -1 & -1 & -1 & 5 \end{pmatrix} \begin{pmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \end{pmatrix} = \begin{pmatrix} 3/2 \\ 3/2 \\ 3/2 \\ 1/2 \\ 3/2 \\ -1/2 \end{pmatrix}$$

(b) $$\begin{pmatrix} 5 & -1 & -1 & -1 & 0 & 0 \\ -1 & 5 & 0 & -1 & -1 & 0 \\ -1 & 0 & 5 & 0 & -1 & -1 \\ -1 & -1 & 0 & 5 & 0 & -1 \\ 0 & -1 & -1 & 0 & 5 & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \end{pmatrix} = \begin{pmatrix} 3/2 \\ 3/2 \\ 3/2 \\ 1/2 \\ 3/2 \\ 0 \end{pmatrix}$$

(c) $$\begin{pmatrix} 3 & -1 & -1 & -1 & 0 & 0 \\ -1 & 3 & 0 & -1 & -1 & 0 \\ -1 & 0 & 3 & 0 & -1 & -1 \\ -1 & -1 & 0 & 3 & 0 & -1 \\ 0 & -1 & -1 & 0 & 3 & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \end{pmatrix} = \begin{pmatrix} 42 \\ 21 \\ 6 \\ -41 \\ 1 \\ 0 \end{pmatrix}$$

(d) $$\begin{pmatrix} 3 & -1 & -1 & -1 & 0 & 0 \\ -1 & 3 & 0 & -1 & -1 & 0 \\ -1 & 0 & 3 & 0 & -1 & -1 \\ -1 & -1 & 0 & 3 & 0 & -1 \\ 0 & -1 & -1 & 0 & 3 & -1 \\ 0 & 0 & -1 & -1 & -1 & 3 \end{pmatrix} \begin{pmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \end{pmatrix} = \begin{pmatrix} 42 \\ 21 \\ 6 \\ -41 \\ 1 \\ -29 \end{pmatrix}$$

(e) None of the above

Name: ______________________________

Instructor: __________________________

6.(6 pts.) An experiment consists of flipping a coin until a tail appears. As soon as a tail appears, the experimenter stops and records the sequence of heads and tails. What is the probability that the outcome of this experiment is $HHHHHHT$?

(a) $\frac{1}{12}$ (b) $\frac{1}{64}$ (c) $\frac{1}{4}$ (d) $\frac{1}{128}$ (e) 0

Name: ______________________________

Instructor: __________________________

7.(6 pts.) If a basketball player attempts 250 shots in a row with a 50% chance of making a basket on each shot, what is the longest run of baskets you would expect in the sequence of outcomes, based on probability.

(a) 8 (b) 6 (c) 7 (d) 4 (e) 10

Name: ______________________________

Instructor: __________________________

Partial Credit

You must show your work on the partial credit problems to receive credit!

8.(10 pts.) There are five dorms competing in the Notre Dame Spring Olympics. There are 7 events; Sumo Wrestling, Polo, Jai Alai, Archery, Raft Racing, Baseball and Basketball. Each dorm has participants in each event. The table below shows the results for 2013 for the six events (with first in an event denoted by 1). It also shows the Borda count for each dorm.

Number of Events

	1	**2**	**1**	**2**	**1**	**Borda Count**
Dillon Hall	4	3	5	3	4	17
Knott Hall	2	1	2	5	3	23
Lewis Hall	3	2	4	4	5	18
McGlinn Hall	5	4	3	1	2	22
Carroll Hall	1	5	1	2	1	25

Each year "The Olympic Cup" is awarded one of the dorms based on their overall performance. If a Condorcet winner exists, the Olympic cup is awarded to that dorm. If not Nanson's method is used to complete the process.

(a) Show that there is no Condorcet winner for the above tournament.

(b) Apply **Nanson's method** to find the winner of the olympic cup?

Name: ____________________________

Instructor: _________________________

9.(10 pts.)

$$\begin{aligned} x + 5y + z &= 8 \\ 2x + 15y + z &= 20 \\ x + y + 2z &= 5 \end{aligned}$$

(a) Write the above system of equations as a matrix equation.

(b) Write the following system of equations as a matrix equation $AX = B$.

$$\begin{aligned} x + 2y &= 3 \\ x + y &= 2 \end{aligned}$$

(c) Solve the system in part (b) by finding the matrix A^{-1} and multiplying the equation by A^{-1}. (show your work for credit).

Name: ___________________________

Instructor: ________________________

10.(10 pts.) (A) Which of the following describes the property of independence from irrelevant alternatives in a voting system?

(a) There are no restrictions placed on the ranking of the candidates a voter may choose.

(b) If all voters prefer candidate A to candidate B, then the group choice should not prefer candidate B to candidate A.

(c) No one individual voter preference totally determines the group choice.

(d) If a group of voters choose candidate A over candidate B,
then the addition or subtraction of other candidates should not change the group choice to B.

(e) If choice A is the winner of an election and, in a reelection, the only changes in the ballots are changes that only favor A, then A should remain the winner of the election.

.

(B) If there are 15 teams in a round robin tournament, how many games must be played?

(C) If an experiment has 35equally likely outcomes, what probability should be assigned to each?

(D) if an experiment consists of flipping a coin 10 times in a row, what is the probability that all of the outcomes will be tails?

Name: ____________________________

Instructor: ________________________

11.(10 pts.) This problem appears as Problem 1 on the take home part of the exam. You may use this page for rough work.

Name: ____________________________

Instructor: ________________________

12.(18 pts.) This problem appears as Problem 2 on the take home part of the exam. You may use this page for rough work.